

# Electroweak phase transition and Higgs self-couplings in the two-Higgs-doublet model

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We calculate both the cubic and the quartic self-couplings of the lighter scalar Higgs boson without assuming the decoupling limit in the two-Higgs-doublet model (THDM). In some regions of parameter space of the THDM where the electroweak phase transition is strongly first order, it is possible that the quartic self-coupling of the lighter scalar Higgs boson might be deviated by at least 40 % from the standard model prediction.

## I. Introduction

Explaining the observed baryon asymmetry of the universe is regarded as one of the basic features for theoretical models to be phenomenologically realistic. Several decades ago, Sakharov suggested three essential conditions for dynamically generating the baryon asymmetry: the violation of baryon number conservation, the violation of both C and CP, and the deviation from thermal equilibrium [1]. Among various mechanisms for explaining the baryon asymmetry, many attentions have been paid to the baryogenesis via the electroweak phase transition (EWPT) [2], which in principle may satisfy the three Sakharov conditions. As is well known, in order to ensure sufficient deviation from thermal equilibrium, the EWPT should be first order, and its strength should be strong, since otherwise the baryon asymmetry generated during the phase transition subsequently would disappear. In general, the phase transition is regarded as a strongly first order if the critical value of the vacuum expectation value of the Higgs field at the broken state is larger than the critical temperature.

The Standard Model (SM), has been investigated whether it can realize the EWPT. It is found, however, that the EWPT faces severe difficulty to be realized in the SM, because the strength of the EWPT is too weak in the SM for the present experimental lower bound on the mass of the Higgs boson. Thus, in the SM, the EWPT is weakly first order or higher order for the experimentally allowed mass of the Higgs boson [3]. Also, the Cabibbo-Kobayashi-Maskawa matrix in the SM [4] cannot produce large enough CP violating phase for generation of baryon asymmetry. Thus, the idea of baryogenesis via the

EWPT requires extensions of the SM to be realized. Many scenarios have been studied in the literature [5-8].

Among them is the two-Higgs-doublet model (THDM) [9]. The presence of an additional Higgs doublet in the THDM enables either explicit or spontaneous CP violation to occur in the Higgs sector of the THDM [10]. Also, it has been observed that there are parameter regions in the THDM where the EWPT is strongly first order to generate the desired baryon asymmetry. Quite recently, phenomenological implications of the THDM within the context of the EWPT have been considered by Okada and his colleagues [11]. In particular, they have suggested that the cubic self-coupling of the lighter scalar Higgs boson in the THDM might be considerably different from the SM prediction. This information would be very useful and duly tested at the future  $e^+e^-$  ILC. In fact, it has already been addressed that the knowledge of both the cubic and the quartic self-couplings of the Higgs bosons is essential for the reconstruction of the necessary self-interaction Higgs potential [12].

We are motivated by the article by Okada and his colleagues, and we would like to study in more detail the self-couplings of the Higgs bosons. In particular, we examine if the quartic self-coupling of the lighter scalar Higgs boson in the THDM might be significantly different from the SM prediction, as well as the cubic self-coupling, by studying the finite temperature effective Higgs potential in the THDM at the one-loop level, without decoupling limit, under the condition of the strongly first order EWPT. We find that not only the cubic self-coupling but also the quartic self-coupling of the Higgs bosons in the THDM exhibits a large devi-

ation from the SM predictions, in the parameter regions where strongly first order EWPT is possible, for values of the lighter scalar Higgs boson mass between 120 and 210 GeV.

## II. The Higgs sector without decoupling limit

Following the notations of Ref. [11], the most general form of the Higgs potential of the THDM at the tree level is given in terms of two Higgs doublets,  $\Phi_1$  and  $\Phi_2$ , by

$$V_{\text{tree}} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_3^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right], \quad (1)$$

where  $\lambda_i$  are quartic couplings and  $m_i$  ( $i = 1, 2, 3$ ) are the mass parameters. The discrete  $Z_2$  symmetry is softly broken by the term proportional to  $m_3^2$ , which prevents the flavor changing neutral current process in the tree level. Assuming no CP violation in the Higgs sector of the THDM, we have five physical Higgs bosons with definite CP after electroweak symmetry breaking: Two neutral scalar Higgs bosons ( $h, H$ ), one neutral pseudoscalar Higgs boson ( $A$ ), and a pair of charged Higgs bosons ( $H^\pm$ ). It is understood that  $h$  is lighter than  $H$ .

We take  $m_1 = m_2 = m$  and  $\lambda_1 = \lambda_2 = \lambda'$  [11], which reduces the field direction relevant to the electroweak phase transition to  $\langle \Phi_1 \rangle = \langle \Phi_2 \rangle = (0, \varphi/2)$ , which corresponds to  $\sin(\alpha - \beta) = -1$  and  $\tan \beta = 1$ , where  $\tan \beta$  is the ratio of the vacuum expectation values  $v_2$  of  $\Phi_2^0$  to  $v_1$  of  $\Phi_1^0$  and  $\alpha$  is the mixing angle between the two scalar Higgs bosons. The vacuum expectation values satisfy  $v = \sqrt{2(v_1^2 + v_2^2)} = 246$  GeV. In terms of  $\varphi$ , the tree-level Higgs potential is given by

$$V_0(\varphi) = -\frac{\mu^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4, \quad (2)$$

where  $\mu^2 = m_3^2 - m^2$  and  $\lambda = (\lambda' + \lambda_3 + \lambda_4 + \lambda_5)/4$ . The tree-level masses of Higgs bosons,  $h, H, A$ , and  $H^\pm$ , are given as  $m_h^2 = 2\lambda v^2$ , and  $m_\phi^2 = M^2 + \lambda_\phi v^2$  ( $\phi = A, H, H^\pm$ ), where  $M^2 = 2m_3^2/\sin 2\beta$  and  $\lambda_\phi$  are the linear combinations of  $\lambda_1$ - $\lambda_5$ . In the decoupling limit, where  $M^2$  is very larger than  $v^2$ ,  $h$  behaves as the SM Higgs boson and the masses of the heavier Higgs bosons are dominantly dependent on  $M$ . We are interested in the non-decoupling limit,

where  $M^2$  is not so large,  $h$  might behave differently from the SM Higgs boson, and the masses of the heavier Higgs bosons are at most a few hundred GeV. Nevertheless, we set  $m_\phi = m_A = m_H = m_{H^\pm}$  for the heavy Higgs boson masses, for simplicity.

The one-loop effective potential at zero temperature  $V_1(\varphi, 0)$  is obtained from the effective potential method as [13]

$$V_1(\varphi, 0) = \sum_l \frac{n_l}{64\pi^2} \left[ m_l^4(\varphi) \log \left( \frac{m_l^2(\varphi)}{m_l^2(v)} \right) - \frac{3}{2} m_l^4(\varphi) + 2m_l^2(v)m_l^2(\varphi) \right], \quad (3)$$

where  $l$  stands for various participating particles: the gauge bosons  $W, Z$ , the third generation quarks  $t, b$ , and the Higgs bosons  $h, H, A, H^\pm$ . The degrees of freedom for each particle are:  $n_W = 6$ ,  $n_Z = 3$ ,  $n_t = n_b = -12$ ,  $n_h = n_H = n_A = 1$ , and  $n_{H^\pm} = 2$ .

The finite-temperature contribution at the one-loop level to the Higgs potential is given by [14]

$$V_1(\varphi, T) = \sum_{l=B,F} \frac{n_l T^4}{2\pi^2} \int_0^\infty dx x^2 \log [1 \pm \exp \left( -\sqrt{x^2 + m_l^2(\varphi)/T^2} \right)], \quad (4)$$

where the negative sign is for bosons ( $B$ ) and the positive sign for fermions ( $F$ ).

One may employ the high temperature approximation to obtain an analytical expression of  $V_1(\varphi, T)$  for qualitative discussions on electroweak phase transition. It is known that in the SM the high temperature approximation is consistent with the exact calculation of the integrals within 5 % at temperature  $T$  for  $m_F/T < 1.6$  and  $m_B/T < 2.2$ , where  $m_F$  and  $m_B$  are the mass of the relevant fermion and boson, respectively. Explicitly, in the high temperature approximation,  $V_1(\varphi, T)$  may be expressed as

$$V_1(\varphi, T) \simeq (DT^2 - E)\varphi^2 - FT\varphi^3 + G\varphi^4, \quad (5)$$

where

$$\begin{aligned} D &= \frac{1}{24v^2} \left( \sum_B n_B m_B^2 + 6m_t^2 + 6m_b^2 \right), \\ E &= \frac{m_h^2}{4} - \frac{1}{32\pi^2 v^2} \left( \sum_{l=B,F} n_l m_l^4 \right), \\ F &= \frac{1}{12\pi v^3} \left( \sum_B n_B m_B^3 \right), \\ G &= \frac{m_h^2}{8v^2} - \frac{1}{64\pi^2 v^4} \left[ \sum_{l=B,F} n_l \log \frac{m_l^2}{a_l T^2} \right], \end{aligned} \quad (6)$$

with  $\log(a_F) = 1.14$  and  $\log(a_B) = 3.91$ . Also we note that the above one-loop effective potential contains the lighter scalar Higgs boson contribution. As one can see from the above expressions, the first order electroweak phase transition is strengthened by the term proportional to  $F$  due to the heavier Higgs boson contributions. If the contributions of heavier Higgs bosons be negligible,  $V_1(\varphi, T)$  would reduce to contain the contribution of only  $h$  in the Higgs sector, thus would become approximately equivalent to the SM. In this case, the electroweak phase transition is either weakly first order or higher order. We perform the exact calculation of the integrals in  $V_1(\varphi, T)$  instead of employing the high temperature approximation.

The full effective potential at finite temperature at one-loop level may now be expressed as

$$V(\varphi, T) = V_0(\varphi, 0) + V_1(\varphi, 0) + V_1(\varphi, T).$$

We emphasize that the above one-loop effective potential contains the contribution of the lighter scalar Higgs boson.

At the one-loop level, the cubic and the quartic self-couplings of the scalar Higgs boson in the SM are respectively given by

$$\begin{aligned}\lambda_{hhh}^{\text{SM}} &\simeq \frac{3m_h^2}{v} \left[ 1 + \sum_l \frac{n_l m_l^4}{12\pi^2 v^2 m_h^2} \right], \\ \lambda_{hhhh}^{\text{SM}} &\simeq \frac{3m_h^2}{v^2} \left[ 1 + \sum_l \frac{4n_l m_l^4}{12\pi^2 v^2 m_h^2} \right],\end{aligned}\quad (7)$$

where  $l$  stands for the gauge bosons, the third generation quarks, and the SM Higgs boson. On the other hand, the cubic and the quartic self-couplings of the lighter scalar Higgs boson in the THDM at the one-loop level are respectively given by

$$\begin{aligned}\lambda_{hhh}^{\text{THDM}} &\simeq \lambda_{hhh}^{\text{SM}} + \frac{m_\phi^4}{\pi^2 v^3} \left( 1 - \frac{M^2}{m_\phi^2} \right)^3, \\ \lambda_{hhhh}^{\text{THDM}} &\simeq \lambda_{hhhh}^{\text{SM}} + \frac{m_\phi^4}{2\pi^2 v^4} \left( 1 - \frac{M^2}{m_\phi^2} \right)^3 \\ &\quad \times \left( 2 + \frac{M^2}{m_\phi^2} \right),\end{aligned}\quad (8)$$

where the contributions of heavier Higgs bosons ( $\phi = H, A, H^\pm$ ) can be collected into one term since  $m_\phi = m_H = m_A = m_{H^\pm}$ .

### III. Numerical Analysis

Now, we define the deviations of the cubic and the quartic self-couplings of the lighter scalar Higgs bo-

son in the THDM from those in the SM Higgs boson, respectively, as

$$\begin{aligned}\Delta_{hhh} &= (\lambda_{hhh}^{\text{THDM}} - \lambda_{hhh}^{\text{SM}}) / \lambda_{hhh}^{\text{SM}}, \\ \Delta_{hhhh} &= (\lambda_{hhhh}^{\text{THDM}} - \lambda_{hhhh}^{\text{SM}}) / \lambda_{hhhh}^{\text{SM}}.\end{aligned}\quad (9)$$

For numerical analysis, we set  $m_W = 80.425$  GeV,  $m_Z = 91.187$  GeV,  $m_t = 174.3$  GeV, and  $m_b = 4.2$  GeV. The remaining free parameters are  $m_h$ ,  $M$ , and  $m_\phi$ .

For  $m_h = 120$  GeV, we examine  $V(\varphi, T)$  for  $0 \leq M \leq 160$  GeV and  $150 \leq m_\phi \leq 400$  GeV, by adjusting the temperature  $T$  to the critical temperature  $T_c$ . We find that  $T_c = 120.7$  GeV for  $m_h = 120$  GeV. We establish the contour of  $v_c/T_c = 1$  in the  $(M, m_\phi)$ -plane, where  $v_c$  is defined as the distance between the two degenerate vacua at  $T_c$ . Our result is shown in Fig. 1 as the solid curve of Set 1. Above the solid curve (the region of larger  $m_\phi$  values) is the region where a strongly first-order EWPT is allowed. We also calculate  $\Delta_{hhh}$  and  $\Delta_{hhhh}$  for the same  $m_h = 120$  GeV, and plot the result in Fig. 1. The contour of  $\Delta_{hhh} = 6\%$ , and that of  $\Delta_{hhhh} = 43\%$  are shown respectively as the dashed curve and the dotted curve of Set 1. As one can see, both contours lie below the contour of  $v_c/T_c = 1$  for the whole parameter space in the  $(M, m_\phi)$ -plane. This implies that it is possible for some region of parameter space of the THDM where the EWPT is strongly first-order that either  $\Delta_{hhh} \geq 6\%$  and/or  $\Delta_{hhhh} \geq 43\%$ . In other words, the THDM may allow a significant deviation from the SM when the EWPT is strongly first-order. Our study of  $\Delta_{hhh}$  for  $m_h = 120$  GeV is consistent with the result of Ref. [11]. On the other hand, the result of  $\Delta_{hhhh}$  is new.

Now, we repeat the analysis for some different values of  $m_h$ . Our results are shown in Fig. 1 as Set 2, 3, and 4. The relevant numbers of Set 2 are:  $m_h = 150$  GeV,  $T_c = 134.8$  GeV,  $\Delta_{hhh} = 7.5\%$ , and  $\Delta_{hhhh} = 44\%$ ; those of Set 3 are:  $m_h = 180$  GeV,  $T_c = 147.4$  GeV,  $\Delta_{hhh} = 10\%$ , and  $\Delta_{hhhh} = 55\%$ ; and those of Set 4 are:  $m_h = 210$  GeV,  $T_c = 159.0$  GeV,  $\Delta_{hhh} = 13\%$ , and  $\Delta_{hhhh} = 62\%$ .

### IV. Conclusions

We find that the results of our study are consistent with the suggestions made by Okada and his colleagues. They have found that the THDM allows a strongly first-order EWPT for successful baryogenesis of the universe for some parameter region,

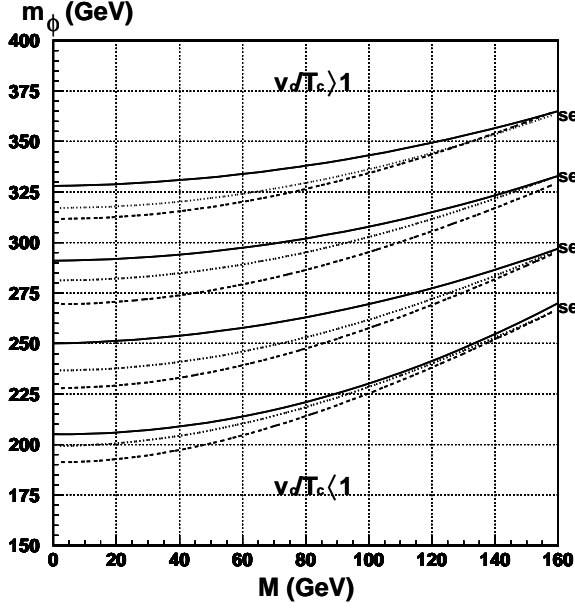


Fig. 1: Contours of  $v_c/T_c = 1$  (solid curve),  $\Delta_{hhh}$  (dashed curve), and  $\Delta_{hhhh}$  (dotted curve), for  $m_h = 120$  GeV (Set 1),  $m_h = 150$  GeV (Set 2),  $m_h = 180$  GeV (Set 3), and  $m_h = 210$  GeV (Set 4) in the  $(M, m_\phi)$ -plane. The values of  $\Delta_{hhh}$  and  $\Delta_{hhhh}$  in each Set are different. See the text.

where the cubic self-coupling of the lighter scalar Higgs boson might be significantly affected. Motivated by the article by Okada and his colleagues, we have studied not only the cubic self-coupling but also the quartic self-coupling of the lighter scalar Higgs boson in the THDM.

We have explored a wide range of value of the lighter scalar Higgs boson mass. Our calculations have been done under the assumption of no CP violation, non-decoupling limit, with reasonable simplifications in relevant parameter values. The results are quite remarkable, especially in the case of the quartic self-coupling. If the mass of the lighter scalar Higgs boson is 210 GeV, it is possible that the quartic self-coupling of the lighter scalar Higgs boson in the THDM may be larger than that of the SM by more than 60 % for some parameter values where the EWPT is strongly first-order. It is found that the cubic self-coupling of the lighter scalar Higgs boson does not exhibit its deviation from the SM as vivid as the quartic self-coupling does. The cubic self-coupling of the lighter scalar Higgs boson in the THDM may be larger than that of the SM by about 13 % if the mass of the lighter scalar Higgs boson

is 210 GeV. For smaller mass of the lighter scalar Higgs boson, the possible magnitude of the deviation in the quartic self-coupling becomes small, but never negligible. We suggest that the deviation of the self-couplings in the THDM from that of the SM, which is induced by the non-decoupling effects of the loops of heavier Higgs bosons at the one-loop level, might provide some basis for the THDM to be further investigated at the future International Linear Collider, ILC.

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